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SHORT COMMUNICATION

STONE COVER ON DESERT HILLSLOPES: EXTENT OF BIAS IN DIAMETERS ESTIMATED FROM GRID SAMPLES AND PROCEDURES FOR BIAS CORRECTION

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ABSTRACT

Grid-based samples of surface stone covers are widely employed for the determination of grain properties such as mean diameter. However, this method has an inherent bias that is related to clast size. Studies failing to correct for this effect have inadvertently reported stone diameters weighted by the area that each stone exposed on the sampling plane.

Monte Carlo methods are used to generate and sample synthetic veneers of surface stones, like those found on many dryland hillslopes, but having pre-determined population distribution characteristics. Uncorrected grid samples from such stone veneers are shown to yield estimates of mean diameter that are in error by up to several hundred per cent. Formulae are provided which permit accuracy of a few per cent to be obtained with samples of 100–300 stones. Slightly larger samples are required accurately to estimate edge length- and area-weighted mean diameters than for traditional volume-weighted means.

KEY WORDS stone cover; grid sampling; grain diameter; sediment size analysis

INTRODUCTION

Studies of slope erosion processes and slope hydrology often require that a population of surface stones be described quantitatively, since stones affect infiltration and runoff production (e.g. Poesen and Lavee, 1994). Commonly, a measure of the grain diameter is used. This is often estimated from a grid sample. A square or rectangular grid is laid out, and 25–200 grains lying at nodes on the grid are measured.

Wolman (1954) advocated grid sampling of river gravels, concluding that by collecting 100 particles, a relatively unbiased estimate of median grain diameter could be obtained. This sample size was not determined on statistical grounds alone; considerations of convenience and simplification of data reduction in the field were also involved. Subsequently, many workers (e.g. Van der Plas, 1962) drew attention to the lack of comparability of the resulting data with those from other methods such as weight-based sieving. In the case of river gravels, Leopold (1970) pointed out that the grid sampling method contains a bias toward larger stones, which have a likelihood of lying beneath a grid node that is disproportionate to their true abundance in the sediment.

He proposed that the raw data be converted to a form more comparable with conventional weight-based sieve data. This was achieved by dividing the estimated total weight of a size category (found by multiplying the number of clasts in that category by an estimated average weight) by the square of the (geometric) mean diameter of that class. This reduces the weight in each class by the reciprocal of the average exposed area of stones in that class. However, interconversions between sampling methods like this have been debated (e.g.

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Kellerhals and Bray, 1971; Diplas and Fripp, 1992). Kellerhals and Bray (1971, p.1172) rejected the data conversion method proposed by Leopold (1970). The alternative conversions proposed by Kellerhals and Bray (1971) were in turn questioned by Diplas and Sutherland (1987) but supported by Church *et al.* (1987).

An analysis of these procedures is important in work on stony soils. Uncorrected grid data may be seriously in error. Further, the bias that arises in grid sampling is not a constant. Rather, it depends on the properties of the material being sampled, especially the degree of sorting. Topographic trends in granulometry may be partially concealed by trends arising from sampling bias. In some studies, no procedure for removing the bias in grid sampling has been adopted, so that the data are difficult to interpret. The studies cited in Table I illustrate the widespread use of grid sampling in the analysis of stone mantles on dryland hillslopes. In none of these studies was an appropriate correction applied to remove the bias inherent in the grid-based stone samples.

The surface stone mantles typical of dryland hillslopes are the focus of the present work. Because the stones on dryland slopes are often only a veneer over finer-textured materials, true volumetric sampling is not possible and any method that is adopted must be surface-based. Grid sampling thus appears attractive, but results presented here suggest that the method is only suitable if adequate bias correction is applied.

METHODS

The investigation used Monte Carlo simulations of surface stone cover. These were selected in preference to field data so that the true population parameters could be known, permitting checks on grid sample accuracy. A hypothetical 3 m × 3 m field plot was established. Within this, random numbers were used to form (x,y) coordinates, and a circular 'stone' located with its centre at each point. Because stone size number distributions from dryland soils show great positive skew, the random numbers used to select stone diameter were drawn from an approximately log normal population whose mean and variance were predetermined. The minimum stone diameter was set at 2 mm to correspond with the minimum size normally measured in the field. Stones were permitted to touch but not overlap. The procedure was iterated until a prespecified

Table I. Details of some particle size investigations in dryland environments based upon grid sampling

Reference	Environment	Sample size and restrictions	Grid dimensions	Grain size statistic adopted
Dury (1966)	Desert hillslope, Australia	50–100 particles	Paced; dimensions not quantified	Uncorrected median *b*-axis
Pérez (1986)	Talus, Venezuela	25 particles >25 mm	5 cm × 5 cm	Uncorrected arithemetic mean *a*-axis
Cooke and Reeves (1972)	Slope debris, Mojave Desert	50 particles >5 mm	50·8 mm × 50·8 mm (2″ × 2″)	Uncorrected arithmetic mean *b*-axis
McFadden *et al.* (1989)	Alluvial fans, Mojave Desert (subsurface)	50 particles >8 mm	Unspecified	Uncorrected arithmetic mean *b*-axis
Abrahams *et al.* (1986)	Desert hillslope, Arizona	76–86 particles	Unspecified	Uncorrected arithmetic mean diameter
Abrahams and Parson (1991)	Desert pavement, Arizona	56–70 particles >2 mm diameter	Unspecified	Uncorrected arithmetic mean *b*-axis
Abrahams *et al.* (1994)	Desert hillslope, Arizona	50 particles >2 mm diameter	5 cm × 10 cm (long axis downslope)	Uncorrected arithmetic mean *b*-axis

areal stone cover fraction was reached, at which point the arrays of (x,y) coordinates, and the stone diameters, were recorded. The arithmetic mean of the population of grain diameters was set to approximately 15 mm, and two levels of sorting were generated, with standard deviations of 10 and 15 mm. Finally, six different cover percentages were generated: 5, 10, 15, 20, 25 and 30 per cent. Individual simulations involved the plotting of up to 8000 stones. A typical simulated stone cover is shown in Figure 1.

Various weighted and unweighted mean diameters were calculated for the entire population of stones set out in a simulation. Grid samples were then taken by scanning the arrays for stones which lay beneath the nodes of a series of square sampling grids. Ten grid spacings were tested: 50, 30, 20, 15, 10, 7·5, 5, 4, 3 and 2 cm. These spaces correspond to grid sample sizes of 49, 121, 256, 441, 961, 1681, 3721, 5776, 10 201 and 22 801 grid nodes. In most cases even the finest grid spacing exceeded the arithmetic mean stone diameter, but in some cases this diameter lay in the 2–3 cm range and was thus spanned by the two finest grids.

Grid samples were collected by sampling with and without replacement. Both methods have been employed in the literature.

The grain diameters assessed are set out in Table II.

Data Processing

Correction of the grid sampling bias was performed on the ungrouped stone diameter data, rather than on data grouped into size classes as is commonly done with sieving, to avoid information loss.

Following Leopold (1970), a weighting was applied to the frequency of each grain sampled from a grid node. This weighting was $1/(d^2)$ where d is the diameter of the grain. Had data been assembled into groups by diameter, the weighting would have been applied to the frequency (or weight) of stones in each size category, as was proposed by Leopold (1970). Again, unbiased data would be obtained but, because of the loss of resolution, accuracy would suffer. This is a particular problem with hillslope stone data, which often display extreme positive skewness, and for which means are prone to disturbance by the shifting of the apparent diameter of a rare large stone when this is allocated to the mid-point of a grain size class rather than retaining the correct absolute diameter.

For the other mean stone diameters of concern here, the formulae which incorporate a correction for

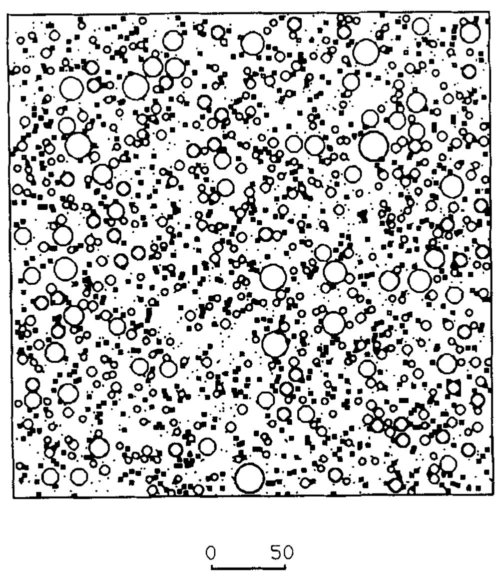

Figure 1. Plot of the simulated stone cover having 20 per cent areal cover and 15 mm target standard deviation

Table II. Formulae employed in calculating mean diameters free of sampling bias (right-hand column) together with the corresponding formulae for samples derived without bias

Parameter	Formula for unbiased data	Formula for biased grid-based samples
Arithmetic mean diameter	$\bar{x}=\frac{\sum_{i=1}^{n} d_i}{n}$	$\bar{x}=\frac{\sum_{i=1}^{n}\frac{1}{d_i}}{\sum_{i=1}^{n}\frac{1}{(d_i)^2}}$
Geometric mean diameter	$\log\bar{x}=\frac{\sum_{i=1}^{n}\log d_i}{n}$	$\log\bar{x}=\frac{\sum_{i=1}^{n}\log d_i\frac{1}{(d_i)^2}}{\sum_{i=1}^{n}\frac{1}{(d_i)^2}}$
Perimeter-weighted mean diameter	$\bar{x}=\frac{\sum_{i=1}^{n} d_i^2}{\sum_{i=1}^{n} d_i}$	$\bar{x}=\frac{n}{\sum_{i=1}^{n}\frac{1}{d_i}}$
Area-weighted mean diameter	$\bar{x}=\frac{\sum_{i=1}^{n} d_i^3}{\sum_{i=1}^{n} d_i^2}$	$\bar{x}=\frac{\sum_{i=1}^{n} d_i}{n}$
Volume-weighted mean diameter	$\bar{x}=\frac{\sum_{i=1}^{n} d_i^4}{\sum_{i=1}^{n} d_i^3}$	$\bar{x}=\frac{\sum_{i=1}^{n} d_i^2}{\sum_{i=1}^{n} d_i}$

sampling bias used in data processing are listed, together with the ordinary formulae for data from unbiased samples, in Table II.

The size of the error that results from the use of uncorrected formulae is analysed below. The only statistic that can be calculated from the raw grid sample diameters is the area-weighted mean grain diameter. Since the set of grain diameters is already area-weighted, the simple arithmetic mean of those diameters yields the area-weighted mean, which is, conveniently, a hydrologically useful parameter (Dunkerley, 1995). Many of the studies cited in Table I have thus employed area-weighted diameters, which are certainly not comparable with weight-based sieve data, without this being made apparent in their presentations. Results from grid samples processed without correction for inherent bias are presented below to illustrate the kinds of errors that may consequently be contained in published estimates of stone diameters.

RESULTS

Not all results of the grid sampling procedures are set out here because of the volume of data, but trends observed in the results are highlighted.

Mean properties of typical populations are specified in Table III.

Results of sampling without bias correction

Figure 2 presents the statistics resulting from sampling with replacement of the stone population having 30

Table III. Characteristics of the simulated stone cover populations

	Grain Population Parameters						
Cover (%)	Target std dev.	Number of stones	Arithmetic mean (cm)	Std dev. (cm)	Geometic mean	Skewness	Kurtosis
30	1·5	4040	2·09	2·0	1·44	2·9	15·6
	1·0	7645	1·77	1·2	1·46	1·8	5·2
25	1·5	2774	2·22	2·3	1·48	2·9	14·1
	1·0	6087	1·79	1·2	1·47	1·8	5·9
20	1·5	2113	2·30	2·4	1·52	2·5	8·4
	1·0	4576	1·84	1·3	1·49	1·8	5·1
15	1·5	1348	2·41	2·6	1·57	3·2	17·1
	1·0	3183	1·88	1·4	1·51	2·1	7·0
10	1·5	878	2·37	2·7	1·54	3·7	20·6
	1·0	2215	1·87	1·3	1·50	1·8	4·6
5	1·5	178	2·76	5·5	1·50	7·3	64·8
	1·0	961	1·92	1·5	1·53	2·8	12·3

per cent cover and 15 mm standard deviation. No correction for the sampling bias was applied to these data, which are therefore comparable to the kinds of results most commonly presented in the case studies from the literature noted earlier.

The results reveal very substantial errors. For the 15 cm grid, where sample size is about 100 stones (the size conventionally suggested to be adequately large), the arithmetic mean is about 254 per cent too large and the geometric mean about 309 per cent too large. Among the weighted means, the volume-weighted mean is most accurate, being about 60 per cent overestimated, while the area-weighted mean is 106 per cent too large and the perimeter-weighted mean is in excess by 168 per cent.

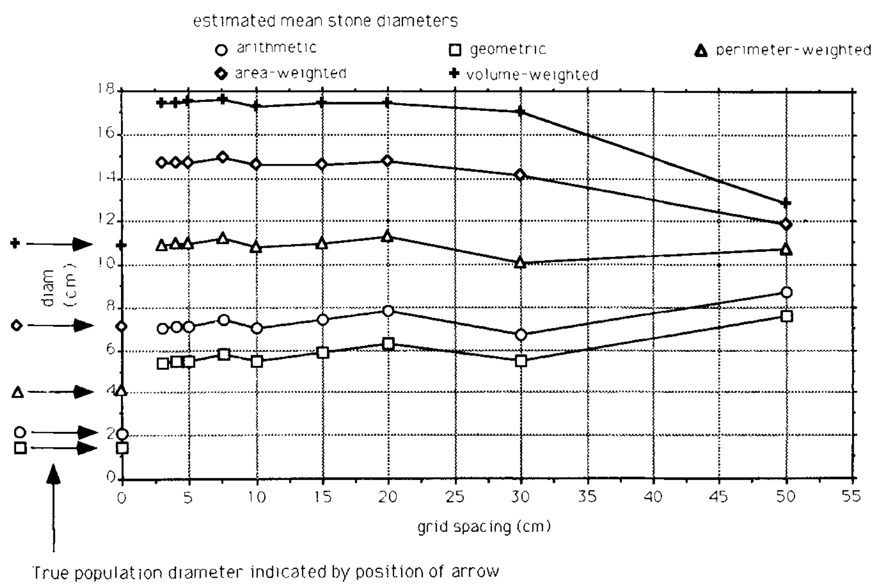

Figure 2. Variation in estimated mean particle diameters with grid spacing, for 30 per cent stone cover with 15 mm standard deviation. Sampling with replacement but with no bias correction applied

An assessment of the dependence of errors on the stone cover fraction and the population standard deviation is presented in Figures 3 and 4. These show the errors in arithmetic mean and geometric mean stone diameter assessed without the removal of sampling bias, for the 3 cm grid. Values at coarser grid spacings are generally little different, and converge toward the value on the finest grid.

It is clear from these plots that errors in estimated mean stone sizes are consistently worse for stone populations which are less well sorted. It is also evident that the error diminishes as stone cover fraction increases, with sample size increasing similarly.

Results for sampling without replacement are not reported here. They are slightly better than for the sampling with replacement, because large stones once sampled cannot be resampled. However, the effect is slight, errors typically only being about 10 per cent smaller than those just reported, and still in the range of 50–300 per cent overestimated.

Statistics with sampling bias removed

Figure 5 presents data for the same stone cover as Figure 2. Again the procedure was sampling with replacement, but with the sampling bias corrected as explained earlier through the use of a $1/d^2$ weighting.

As can be seen from these results, accurate estimates of the population parameters are much more rapidly achieved, errors falling to less than 10 per cent at grid spacings of 15 cm or so, with sample sizes of only approximately 100 stones.

Though additional data would be valuable, it can nevertheless be seen that the often-cited assurance, based largely upon the work of Leopold (1970) and Hey and Thorne (1983), that samples of 40–100 stones suffice to define mean diameters may not be applicable to data of this kind. Volume-weighted means, which are relatively insensitive to small stones unless these are in great abundance, rapidly converge on the population value. The maximum error in the volume-weighted mean is 7·3 per cent for a sample of only 22 stones. Area and perimeter means, however, proportional to lower powers of d, converge less rapidly. The perimeter-weighted mean is the slowest to converge on an accurate value, but even here the error falls to 10 per cent for samples of about 100 stones and is only a little over 7 per cent for a sample size of about 450 stones. The full data set, not presented here owing to its size, reveals that neither variation in the population standard deviation, nor in the cover fraction, greatly affects the magnitude of sampling errors, at least in the ranges tested.

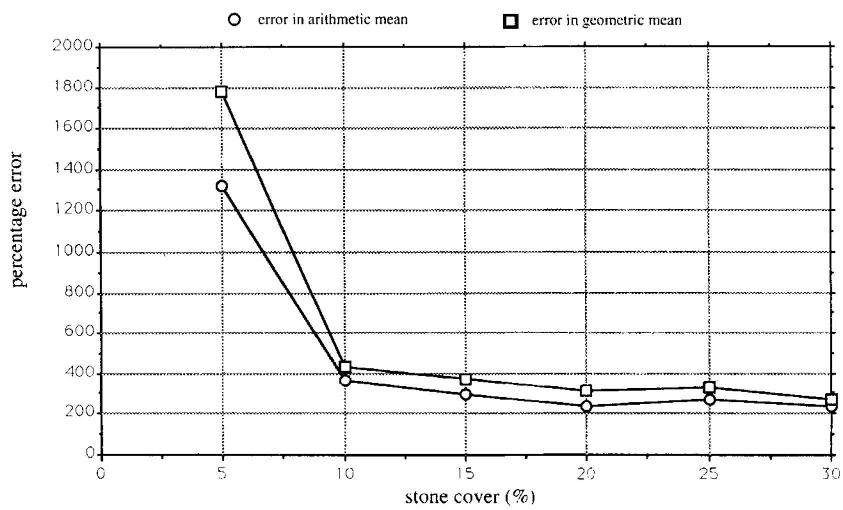

Figure 3. Behaviour of the error in the two simple means derived by sampling with replacement on the 3 cm × 3 cm grid, without bias correction applied, across stone covers of 5–30 per cent with target standard deviation of 15 mm

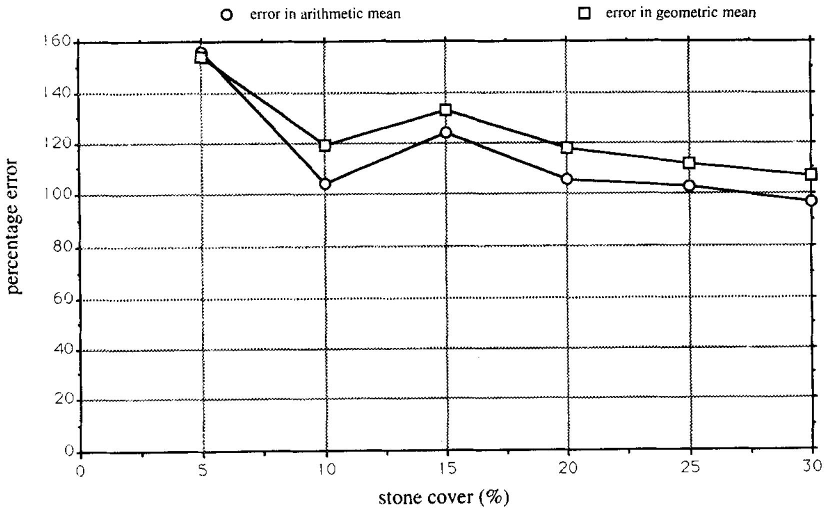

Figure 4. Behaviour of the error in the two simple means derived by sampling with replacement on the 3 cm × 3 cm grid, without bias correction applied, across stone covers of 5–30 per cent with target standard deviation of 10 mm

These results are dramatically better than those from the grid samples processed without bias correction. Less sampling effort is involved, and errors are an order of magnitude smaller.

CONCLUSION

Depending upon the grain size parameter calculated and the particular sampling strategy employed, errors in stone diameter estimated from uncorrected grid samples in dryland environments can range up to 300 per cent

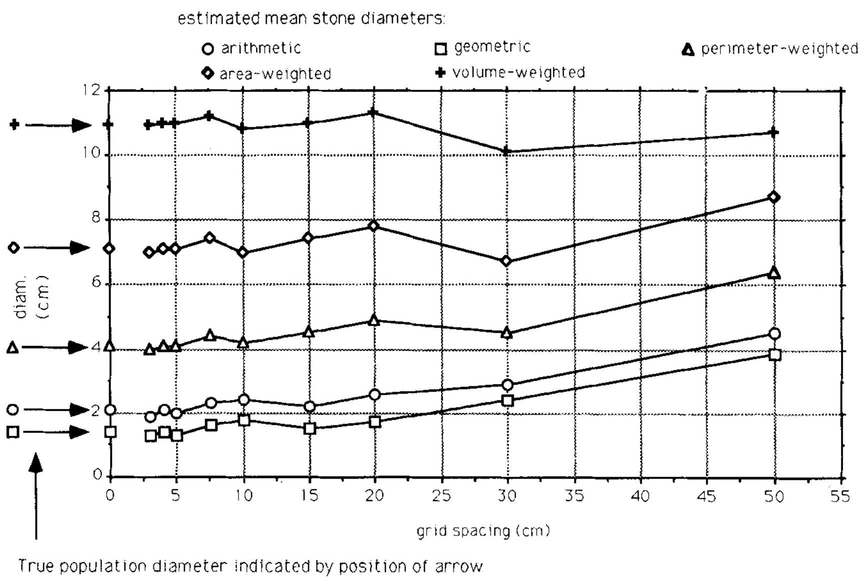

Figure 5. Variation in estimated mean particle diameters with grid spacing, for 30 per cent stone cover with 15 mm standard deviation. Sampling with replacement and with bias correction applied

and more. Worse, if downslope trends in size and sorting are the object of study, true grain size trends will be misrepresented in grid-based data. If, as is commonly the case, stones are smaller and better sorted downslope, then the downslope rate of grain size decline will potentially be greatly overstated. Poorly sorted upslope sites will exhibit a substantial bias toward the larger exposed grains, and grid sampling may indicate a much coarser mean stone size there than is actually the case.

The removal of bias is very straightforward and results in much greater accuracy with less sampling effort. While samples of 100 stones or fewer permit estimation of volume-weighted mean diameters, akin to those achieved by sieve analysis, these are of limited relevance in studies of hillslope hydrology. Rather, mean diameters weighted by stone surface area and edge-length are required. To estimate these parameters, significantly larger sample sizes are required. In the case of the stone cover data shown in Figure 5, sample sizes of several hundred grains are needed to restrict errors to <10 per cent, and larger samples would be needed in more poorly sorted materials. Such samples can be collected conveniently, though with some limitations, from photographs of the stone mantles (Dunkerley, 1995).

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